Recombination limited energy relaxation in a BCS superconductor

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We study quasiparticle energy relaxation at sub-kelvin temperatures by injecting hot electrons into an aluminium island and measuring the energy flux from electrons into phonons both in the superconducting and in the normal state. The data show strong reduction of the flux at low temperatures in the superconducting state, in qualitative agreement with the presented quasiclassical theory for clean superconductors. Quantitatively, the energy flux exceeds that from the theory both in the superconducting and in the normal state, possibly suggesting an enhanced or additional relaxation process.

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Superconducting nanostructures attract lots of attention currently, partly because of their potential applications, for instance, in single Cooper pair and single-electron devices, in quantum information processing, and in detection of radiation. Although the operation of many of these devices is based on charge transport, the energy relaxation in them is also of importance to warrant proper functioning either under driven conditions, or when subjected to environment fluctuations. Thermalization of the electron system with the surrounding bath is a serious concern at sub-kelvin temperatures for non-superconducting structures, but securing proper thermalization of a superconductor is an even greater challenge. In particular, recombination of hot quasiparticles (QP:s) into Cooper pairs slows down exponentially towards low temperatures. QP scattering rates in usual BCS superconductors have been assessed theoretically already several decades ago [1, 2], and there have been measurements of them, both soon after the first predictions (for review, see [1]), and recently also at very low temperatures [3, 4, 5]. However, although a well recognized issue in normal systems, the most relevant property of relaxation, the associated heat flux, has not been addressed in the past. This is the topic of the present letter. We present both experimental and theoretical results which demonstrate the importance of slow thermal relaxation in superconducting nanostructures.

Energy relaxation in normal metals has been investigated thoroughly in experiment for a long time [6, 7, 8, 9]. The central results can be summarized as follows. In three dimensional systems electron-phonon (e-p) heat flux $P_{\rm ep}$ is

$$P_{\rm ep} = \Sigma \mathcal{V}(T_{\rm e}^5 - T_{\rm p}^5). \qquad (1)$$

Here, Σ is a material constant [10], $\mathcal{V}$ is the volume of the electronic system, and $T_{\rm e}$ and $T_{\rm p}$ are the temperatures of electrons and phonons, respectively. Deviations from this behaviour towards the fourth power of temperature have been seen for lower temperatures (see, for example,

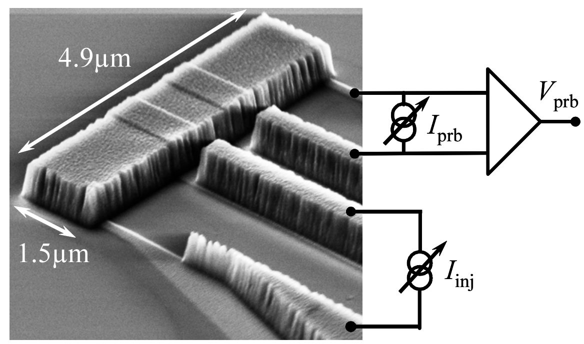

FIG. 1: A typical sample (Sample C) for measuring energy relaxation in an aluminium superconducting bar. The circuits on the right indicate injection of hot QP:s and probing the island temperature.

Ref. [9] and also more recent Ref. [11]), and are usually explained by the impurity effects [12, 13, 14] when the wave length of a thermal phonon becomes of the order of the electron mean free path or of the sample size. Nevertheless, Eq. (1) gives a good account of the heat flux observed in most experiments at sub-kelvin temperatures. Under the same conditions electron-electron (e-e) relaxation is typically much faster; most experiments demonstrate the so-called quasi-equilibrium, where electrons have a well-defined temperature, usually different from that of the phonons. Deviations from this simple picture have been observed, e.g., in voltage biased diffusive wires [15].

Relaxation processes in superconductors have also been studied, see, e.g., Ref. [1]. The most obvious features different from the normal state are: (i) The QP:s need to emit or absorb an energy in excess of the energy gap Δ to be recombined or excited, respectively. This leads to exponentially slow e-p relaxation rates at low temperatures. (ii) The number of QP:s is very small well below T_C, leading to slow e-e relaxation as well. The focus has been in relaxation times, with no attempts to

obtain energy flux in the spirit of Eq. (1). The relaxation time was addressed recently, e.g., in experiments on superconducting photon detectors [3, 4, 5]; these measurements suggest to confirm the recombination limited rate $\tau_{\rm rec}^{-1} \propto \sqrt{T/T_C} e^{-\Delta/k_BT}$ down to $T/T_C \simeq 0.2$. At lower T the relaxation time saturates due to presently poorly known reasons.

The e-p processes in clean superconductors can be characterized by the rate $\tau_{\mathbf{k},\mathbf{k}-\mathbf{q}}^{-1}$ of a QP with wave vector $\mathbf{k}$ to emit a phonon with wave vector $\mathbf{q}$, $\tau_{\mathbf{k},\mathbf{k}-\mathbf{q}}^{-1} = (2\pi/\hbar)|\mathcal{M}_{\mathbf{k},\mathbf{k}-\mathbf{q}}|^2 \delta(E_{\mathbf{k}} - E_{\mathbf{k}-\mathbf{q}} - \epsilon_{\mathbf{q}}) N(E_{\mathbf{k}-\mathbf{q}})[1 - f(E_{\mathbf{k}-\mathbf{q}})][n_p(\mathbf{q}, T_{\rm p}) + 1]$. Here, $\mathcal{M}_{\mathbf{k},\mathbf{k}-\mathbf{q}}$ is the matrix element of electron-phonon coupling in a superconductor, $f(E)$ is the distribution function of electrons; it is the Fermi function $f(E, T_{\rm e}) = (1 + e^{E/k_BT_{\rm e}})^{-1}$ if electrons are in equilibrium with a temperature $T_{\rm e}$. Phonons are assumed to be in equilibrium with occupation $n_p(\mathbf{q}, T_{\rm p}) = (e^{\epsilon_{\mathbf{q}}/k_BT_{\rm p}} - 1)^{-1}$. Compared to the normal state [8], we have inserted the normalized density of states (DOS) $N(E)$. For a superconductor with energy gap $\Delta(T)$, the DOS also depends on temperature $N(E,T) = |E|/\sqrt{E^2 - \Delta(T)^2}\Theta(E^2 - \Delta(T)^2)$, where $\Theta(x)$ is the Heaviside step function. Electrons emit energy to phonons at the rate $P_e = N(E_F) \int dE_{\mathbf{k}} N(E_{\mathbf{k}}) f(E_{\mathbf{k}}, T_{\rm e}) \int d^3q D_p(\mathbf{q}) \epsilon_{\mathbf{q}} \tau_{\mathbf{k},\mathbf{k}-\mathbf{q}}^{-1}$. $D_p(\mathbf{q})$ is the phonon DOS and $N(E_F)$ is the normal-state DOS at the Fermi level. Writing a similar expression for absorption of phonons P_a, we obtain the net heat flux, $P_{\rm ep} = P_e - P_a$. Calculating the rate and the matrix elements from the quasiclassical theory for clean superconductors [16] we find the e-p heat flux

$$P_{\rm ep} = -\frac{\Sigma\mathcal{V}}{96\zeta(5)k_B^5} \int_{-\infty}^{\infty} dE\, E \int_{-\infty}^{\infty} d\epsilon\, \epsilon^2 {\rm sign}(\epsilon) M_{E,E+\epsilon} \times \big[\coth(\frac{\epsilon}{2k_BT_{\rm p}})(f_E^{(1)} - f_{E+\epsilon}^{(1)}) - f_E^{(1)} f_{E+\epsilon}^{(1)} + 1\big]. \quad (2)$$

Here, $f_E^{(1)} = f(-E) - f(E)$; for equilibrium, $f_E^{(1)} = 1 - 2f(E, T_{\rm e}) = \tanh(\frac{E}{2k_BT_{\rm e}})$, and $M_{E,E+\epsilon}$ is given by $M_{E,E'} = N(E)N(E')[1 - \frac{\Delta^2(T_{\rm e})}{EE'}]$. In the regime $T_{\rm p} \ll T_{\rm e} \ll \Delta/k_B$ we obtain $P_{\rm ep} \simeq \frac{64}{63\zeta(5)} \Sigma\mathcal{V} T_{\rm e}^5 e^{-\Delta/k_BT_{\rm e}}$, which is by a factor $0.98 e^{-\Delta/k_BT_{\rm e}}$ smaller than the result for the normal state [Eq. (1) with $T_{\rm p} \ll T_{\rm e}$].

Figure 1 shows a typical configuration of our experiments. The samples were made by electron beam lithography and shadow evaporation. The parameters of the structures are given in Table I. The aluminium block in the centre of Fig. 1 is the volume in which energy relaxation is investigated. Two small and two large tunnel junctions connect the island to aluminium leads. The hot QP:s are injected via one of the small tunnel junctions in series with a large one. Because of the large asymmetry of junction parameters, essentially all the power is injected by the small junction. The steady-state distribution on the island is deduced from the current-voltage curves (IVs) of the opposite pair of junctions. We observe the QP current of only the small junction; the large junction remains in the supercurrent state. Measurements in a configuration with two small junctions in series as injectors and the two large junctions in series as probes were also made with essentially identical results.

TABLE I: Sample dimensions and junction resistances.

Sample	volume (μm^3)	R_1, R_2, R_3, R_4 (kΩ)
A	$21 \cdot 1.5 \cdot 0.44$	840, 4, 4, 1160
B	$4.9 \cdot 1.5 \cdot 0.44$	760, 5.7, 5.7, 1290
C	$4.9 \cdot 1.5 \cdot 0.44$	485, 20, 20, 980

If the island is at temperature $T_{\rm e}$ and the leads at $T_{\rm ext}$, the QP current I is given by $eR_TI = \int dE N(E - eV, T_{\rm e}) N(E, T_{\rm ext})[f(E - eV, T_{\rm e}) - f(E, T_{\rm ext})]$. Here R_T is the normal state resistance of the junction. For equal island and lead temperatures, $T_{\rm e} = T_{\rm ext}$, the calculated (a) and measured (b) IVs for various $T_{\rm e}/T_C$ are shown in Fig. 2. Wide plateaus in the regime $0 < eV < 2\Delta$ emerge due to the thermal QP current; its value at $eV = \Delta$ is shown in (c). The agreement between experiment and theory is good down to $T_{\rm e}/T_C \simeq 0.25$. To match the data to the theory also at lower temperatures one can use the pair-breaking parameter $\gamma \equiv \Gamma/\Delta$ which yields a smeared density of states, $N(E,T) = |{\rm Re}(E + i\Gamma)/\sqrt{(E + i\Gamma)^2 - \Delta(T_{\rm e})^2}|$. In the figure we show lines with $\gamma = 10^{-4}$ and $\gamma = 10^{-3}$. Since at higher temperatures no fit parameter is needed, we focus our analysis to the range $0.3 < T_{\rm e}/T_C < 1$ to be on the safe side.

Figure 2 (d) shows the calculated IVs of the probe junction, assuming that only the island temperature $T_{\rm e}$ is elevated, and the leads remain at $T_{\rm ext} = 0.05T_C$. This is the expected behaviour under power injection, provided the e-e relaxation is strong and that the junctions are opaque enough not to conduct heat from the island into the leads. A peak in the IVs arises at $eV = \Delta(T_{\rm ext}) - \Delta(T_{\rm e})$. In Fig. 2 (e) we show the corresponding measured curves at various levels of injected power. The resemblance between (d) and (e) is obvious but the features in experimental curves are broadened in comparison to those from the theory, which is common for small junctions [17]. In the data analysis we next find the minimum current in the plateau-like regime at bias voltages between the "matching" peak and the strong onset of QP current. This current is converted into temperature by comparing it to the $T_{\rm e}$ dependent minimum current of the theoretical IVs.

The power $\dot{Q}(V)$ deposited on the island by a biased junction is given by $e^2R_T\dot{Q}(V) = \int (E - eV) N(E - eV, T_{\rm e}) N(E, T_{\rm ext})[f(E, T_{\rm ext}) - f(E - eV, T_{\rm e})]dE$. This equation allows us to determine the injected power, as well as the heat flux through all the junctions. There are two features to note: (i) Since typical injection voltages in the experiment are $V \gg \Delta/e$, it is sufficient to assume that the power injected into the island equals $IV/2$, i.e., it is divided evenly between the two sides of the junc-

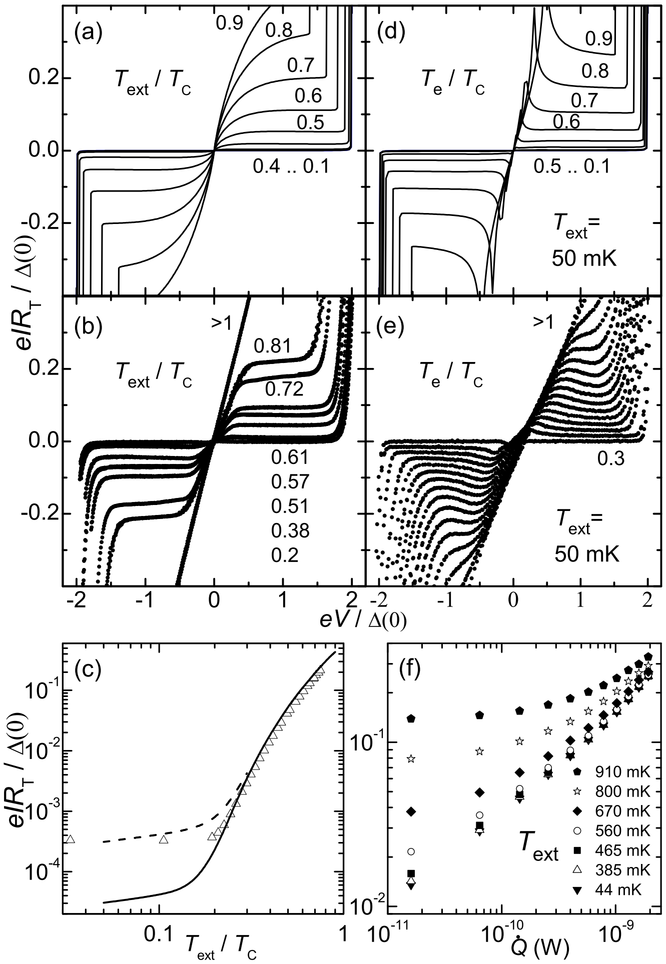

FIG. 2: Tunnel currents under equilibrium and quasi-equilibrium conditions for a superconductor. Theoretical (a) and experimental (b) IVs of a junction at several bath temperatures. (c) Theoretical and experimental currents at $eV = \Delta$. The two theory lines correspond to two different realistic pair breaking parameters $\gamma = 10^{-3}$ (upper curve) and $\gamma = 10^{-4}$ (lower curve). (d) Calculated IVs when the two leads of the junction have different temperatures. (e) The measured IVs under a few injection conditions. (f) The current in Sample A on the plateau between the initial peak and the rise of the current at the conduction threshold around $2\Delta/e$.

tion: the junction behaves essentially as a normal junction, where this statement is true always. (ii) The heat conductance of the (probing) junction is almost constant over a wide range of voltages within the gap region. Its value is low and can be neglected under most experimental conditions. Yet, to test this, we varied the resistances of the large tunnel junctions by a factor of five between samples A and C, without a significant effect on the results. Figure 2 (f) shows the current on the plateau, as described in the previous paragraph, as a function of power injected, at various bath temperatures. In a wide range, from 30 mK up to 380 mK, the behaviour is almost identical: only the higher temperature among $T_{\rm e}$ and $T_{\rm p}$ plays a role, in consistence with the theoretical discussion. Therefore we compare the experimental results at the base phonon temperature (of about 50 mK) to the theoretical results for $T_{\rm p} \ll T_{\rm e}$ in what follows.

We studied $P_{\rm ep}$ in the normal state as well by applying a magnetic field of about 120 mT to suppress the superconductivity and measuring the partial Coulomb blockade (CB) signal [18]. Like in the superconducting state, two regimes are possible. In equilibrium the results of Ref. [18] apply. Under injection, the typical situation is such that $T_{\rm ext} \ll T_{\rm e}$, which we discuss now in more detail. The tunnelling rates in a state with an extra charge n for adding (+) or removing (−) an electron to the normal island with electrostatic energy change $\Delta F^{\pm}(n) = \pm 2E_C(n \pm 1/2) \mp eV/2$ are

$$\Gamma^{\pm}(n) = \frac{1}{e^2R_T}\int_{-\infty}^{\infty} dE\, f_1(E)[1-f_2(E-\Delta F^{\pm}(n))]. \quad (3)$$

Here $E_C = e^2/2C_\Sigma$ is the charging energy of the island with the total capacitance C_Σ, and f_1 and f_2 are the distributions on the source and target electrodes. For equilibrium distribution $f_i(E) = (1+e^{E/k_BT_i})^{-1}$ with $T_1 = T_2$, Eq. (3) yields the result of Ref. [18]. Here we have the opposite limit of low bath temperature, $T_{\rm ext} = T_1 \ll T_2 = T_{\rm e}$. For $T_1 = 0$, $f_1(E) = 1 - \Theta(E)$, yielding $\Gamma^{\pm}(n) = (k_BT/e^2R_T)\ln(1+e^{-\Delta F^{\pm}(n)/k_BT_{\rm e}})$. The current into the island is $I = e\sum_{n=-\infty}^{\infty}\sigma(n)[\Gamma^+(n)-\Gamma^-(n)]$ where $\sigma(n)$ is the probability of having n extra electrons on the island. Since $\sum_{n=-\infty}^{\infty} n\sigma(n) = 0$ by symmetry, and $\sum_{n=-\infty}^{\infty}\sigma(n) = 1$, we find for the differential conductance up to the first order in $E_C/k_BT_{\rm e}$

$$\frac{G^{\rm neq}}{G_T} = 1 - \frac{E_C}{2k_BT_{\rm e}}\frac{1}{\cosh^2(eV/4k_BT_{\rm e})}. \quad (4)$$

The depth of the conductance minimum at $V = 0$ is $\Delta G/G_T = E_C/2k_BT_{\rm e}$ which is 50% larger than that in the equal temperature case. To find the width at half minimum we need to solve $\cosh^2(eV_{\pm}/4k_BT_{\rm e}) = 2$. The full width is $V_{1/2}^{\rm neq} = |V_+ - V_-|$ or $V_{1/2}^{\rm neq} = 4\ln(3+2\sqrt{2})k_BT_{\rm e}/e$. This is about 65% of the equal-temperature value, $V_{1/2}^{\rm eq} \simeq 10.88k_BT_{\rm e}/e$ [18].

Figure 3 is a collection of the data at the base temperature ($\simeq$ 50 mK), in form of island temperature $T_{\rm e}/T_C$ as a function of injected power. The superconducting state was measured for the three samples. The power has been normalized by that at T_C, to present data from different samples on the same footing. For samples A, B and C, $P(T_C)$ = 14 nW, 3 nW, and 3 nW, respectively. The data on the three samples are mutually consistent. The quasiclassical result for a superconductor is shown by a solid line. The normal state data were taken for Sample C which is ideal for a measurement of the island temperature via partial CB: it has $E_C/k_B \simeq 20$ mK yielding an approximately 8% deep Coulomb dip in conductance at

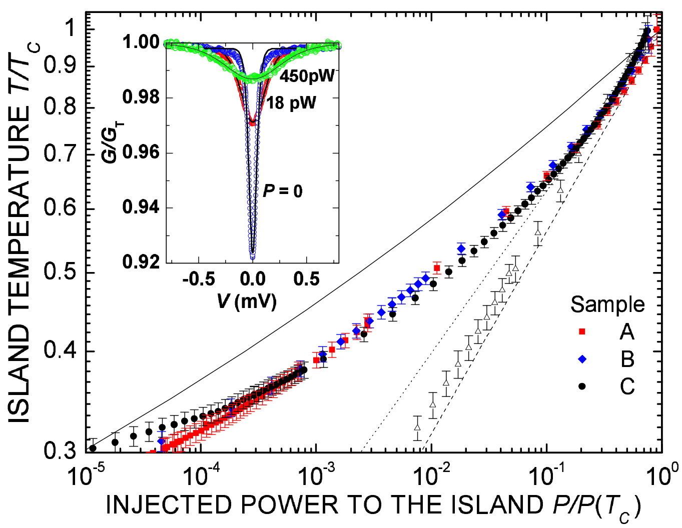

FIG. 3: (color online) Energy relaxation from theory and experiment. The data in the superconducting state are from Sample A (squares), B (diamonds), and C (circles). The open triangles are from Sample C in the normal state. The solid line is the result of Eq. (2) in the superconducting state. The dotted line indicates $P/P(T_C) = (T/T_C)^5$, and the dashed line $P/P(T_C) = (T/T_C)^4$. The inset shows three Coulomb peaks measured in the normal state under different levels of power injection: the solid lines are theoretical fits to them.

zero bias at 90 mK (see the inset of Fig. 3). The two large junctions were used for probing and the small ones for power injection. We first checked that the value $V_{1/2}^{\rm eq}$ yields a good quantitative agreement with the equilibrium temperature data over the whole range of the experiment. Next we measured the quasi-equilibrium electronic temperature under injection. The low base temperature permits the use of the expression of $V_{1/2}^{\rm neq}$ above to extract $T_{\rm e}$ in the range displayed in Fig. 3. Power law type behavior can be observed over the whole temperature range $0.3T_C < T_{\rm e} \lesssim T_C$. The data approach those of the superconducting state near $T_C \simeq 1.45$ K, as expected. The power law for $P_{\rm ep}$ is, however, better approximated by $T_{\rm e}^4$ (dashed line) instead of $T_{\rm e}^5$ (dotted line) of Eq. (3), yielding a deviation of the same sign with respect to the basic theories as in the superconducting state.

The experimental data demonstrate that e-p coupling in a superconductor is weaker than in the normal state, by two orders of magnitude at $T_{\rm e}/T_C = 0.3$. But, like in the relaxation time experiments in a superconductor [3, 4, 5], the energy flux is larger than that from the quasiclassical theory [1, 16]. This observation could suggest that the electron relaxation rate both in the superconducting and in the normal state might be sensitive to the microscopic quality and the impurity content of the particular film [14]. The impurity effects on the e-p relaxation are controlled by the parameter $q\ell$ where ℓ is the electronic mean free path and $q = k_B T_{\rm e}/\hbar u$ is the wave vector of an emitted phonon with energy of the order of the electronic temperature. With the speed of sound $u \sim 5000$ m/s and $\ell \sim 20$ nm in our samples, we have $q\ell \sim 0.5\ \mathrm{K}^{-1}\ T_{\rm e}$. Thus the impurity effects can become essential below 1 K. More theoretical studies are thus needed of the impurity effects on the e-p interaction in the superconducting state.

Our experiments were performed on three samples with very different lengths and junction parameters but they yielded essentially identical results when normalized by the island volume. Therefore we believe that issues like thermal gradients, slow electron-electron relaxation, and the presence of tunnel contacts have only a minor influence on the results. The data thus yield the intrinsic energy relaxation of QP:s in the superconducting and in the normal state. In summary, the experiment follows qualitatively the theoretical model that we presented. On the quantitative level there is a substantial discrepancy especially for superconductors, which would imply that one needs to invoke an extra relaxation channel to account for. Solution of this quantitative disagreement and experiments at still lower temperatures remain as topics of future work.

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